

A Representations of $SU(2)$

In this appendix we provide details of the parameterization of the group $SU(2)$ and differential forms on the group space.

An arbitrary representation of the group $SU(2)$ is given by the set of three generators T_k , which satisfy the Lie algebra

$$[T_i, T_j] = i\varepsilon_{ijk}T_k, \quad \text{with } \varepsilon_{123} = 1.$$

The element of the group is given by the matrix

$$U = \exp \{i\mathbf{T} \cdot \boldsymbol{\omega}\}, \quad (\text{A.1})$$

where in the fundamental representation $T_k = \frac{1}{2}\sigma_k$, $k = 1, 2, 3$, with

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

standard Pauli matrices, which satisfy the relation

$$\sigma_i \sigma_j = \delta_{ij} + i\varepsilon_{ijk}\sigma_k.$$

The vector $\boldsymbol{\omega}$ has components ω_k in a given coordinate frame.

Geometrically, the matrices U are generators of spinor rotations in three-dimensional space \mathbb{R}^3 and the parameters ω_k are the corresponding angles of rotation. The Euler parameterization of an arbitrary matrix of $SU(2)$ transformation is defined in terms of three angles θ , φ and ψ , as

$$\begin{aligned} U(\varphi, \theta, \psi) &= U_z(\varphi)U_y(\theta)U_z(\psi) = e^{i\sigma_3\frac{\varphi}{2}}e^{i\sigma_2\frac{\theta}{2}}e^{i\sigma_3\frac{\psi}{2}} \\ &= \begin{pmatrix} e^{i\frac{\varphi}{2}} & 0 \\ 0 & e^{-i\frac{\varphi}{2}} \end{pmatrix} \begin{pmatrix} \cos \frac{\theta}{2} & \sin \frac{\theta}{2} \\ -\sin \frac{\theta}{2} & \cos \frac{\theta}{2} \end{pmatrix} \begin{pmatrix} e^{i\frac{\psi}{2}} & 0 \\ 0 & e^{-i\frac{\psi}{2}} \end{pmatrix} \\ &= \begin{pmatrix} \cos \frac{\theta}{2} e^{\frac{i}{2}(\psi+\varphi)} & \sin \frac{\theta}{2} e^{-\frac{i}{2}(\psi-\varphi)} \\ -\sin \frac{\theta}{2} e^{\frac{i}{2}(\psi-\varphi)} & \cos \frac{\theta}{2} e^{-\frac{i}{2}(\psi+\varphi)} \end{pmatrix}. \end{aligned} \quad (\text{A.2})$$

Thus, the $SU(2)$ group manifold is isomorphic to three-sphere S^3 . The Euler angles θ , φ and ψ take values within the intervals $0 \leq \theta \leq \pi$, $0 \leq \varphi \leq 2\pi$ and $0 \leq \psi \leq 4\pi$. Note that the reduction to the parametric space of the

orthogonal group $SO(3)$ can be achieved, if we fix the range of values of the angle ψ to be restricted to the interval $0 \leq \psi \leq 2\pi$ and make the identification $\psi \sim \psi + 2\pi$.

Using the matrices (A.2), we can define so-called canonical left and right one-forms on the group $SU(2)$, which are also called *Maurer–Cartan one-forms* (note that $dU^{-1}U = -U^{-1}dU$)

$$R = U^{-1}dU = \frac{i}{2}\sigma_k R_k, \quad L = dU U^{-1} = \frac{i}{2}\sigma_k L_k. \quad (\text{A.3})$$

Since $\det U = 1$, we have the condition on these forms

$$d \det U = d e^{\text{tr} \ln U} = \text{tr} L = \text{tr} R = 0.$$

The components of the Maurer–Cartan forms in the basis given by the Pauli matrices are written as

$$\begin{aligned} R_1 &= -\sin \psi d\theta + \cos \psi \sin \theta d\varphi, & L_1 &= \sin \varphi d\theta - \cos \varphi \sin \theta d\psi, \\ R_2 &= \cos \psi d\theta + \sin \psi \sin \theta d\varphi, & L_2 &= \cos \varphi d\theta + \sin \varphi \sin \theta d\psi, \\ R_3 &= d\psi + \cos \theta d\varphi, & L_3 &= d\varphi + \cos \theta d\psi. \end{aligned} \quad (\text{A.4})$$

Clearly, they satisfy the *Maurer–Cartan equations*

$$dR_n = \frac{1}{2}\varepsilon_{nmk}R_m \wedge R_k, \quad dL_n = -\frac{1}{2}\varepsilon_{nmk}L_m \wedge L_k.$$

In the same way, we can define the set of angular coordinates $\tilde{\psi}, \tilde{\theta}, \tilde{\varphi}$, that parameterizes the sphere $SO(3)$ and the one-forms on the space of parameters of this group.

The left and right forms on the group $SU(2)$ are dual to the vector field ξ_k , components of which form the standard basis of the Lie algebra on the group $SU(2)$:

$$\langle \xi_k^{(R)}, R_m \rangle = \delta_{km}, \quad \langle \xi_k^{(L)}, L_m \rangle = \delta_{km}.$$

Here the right and left Killing vectors are related with generators of rotations about the corresponding axis of Cartesian coordinates. They can be written in terms of the Euler parameterization as

$$\begin{aligned} \xi_1^{(R)} &= -\cot \theta \cos \psi \frac{\partial}{\partial \psi} - \sin \psi \frac{\partial}{\partial \theta} + \frac{\cos \psi}{\sin \theta} \frac{\partial}{\partial \varphi}, \\ \xi_2^{(R)} &= -\cot \theta \sin \psi \frac{\partial}{\partial \psi} + \cos \psi \frac{\partial}{\partial \theta} + \frac{\sin \psi}{\sin \theta} \frac{\partial}{\partial \varphi}, \\ \xi_3^{(R)} &= \frac{\partial}{\partial \psi}, \end{aligned} \quad (\text{A.5})$$

and

$$\begin{aligned}\xi_1^{(L)} &= -\frac{\cos \varphi}{\sin \theta} \frac{\partial}{\partial \psi} + \sin \varphi \frac{\partial}{\partial \theta} + \cot \theta \cos \varphi \frac{\partial}{\partial \varphi}, \\ \xi_2^{(L)} &= \frac{\sin \varphi}{\sin \theta} \frac{\partial}{\partial \psi} + \cos \varphi \frac{\partial}{\partial \theta} - \cot \theta \sin \varphi \frac{\partial}{\partial \varphi}, \\ \xi_3^{(L)} &= \frac{\partial}{\partial \varphi},\end{aligned}\tag{A.6}$$

for the left and right Killing vector field, respectively. The vector fields on the parameter space of the $SO(3)$ group can be constructed in the same way.

Note that the generators of the left and right rotations commute, while left and right Killing vectors satisfy the $SU(2)$ Lie algebra

$$[\xi_m^{(R)}, \xi_n^{(R)}] = -\varepsilon_{mnk} \xi_k^{(R)}, \quad [\xi_m^{(L)}, \xi_n^{(L)}] = \varepsilon_{mnk} \xi_k^{(L)}, \quad [\xi_m^{(L)}, \xi_n^{(R)}] = 0.$$

Thus, the right one-form R_n is invariant with respect to the left action of the $SU(2)$ group while the left one-form L_n is invariant with respect to the right action of the group $SU(2)$, i.e., the corresponding Lie derivative with respect to $\xi_n^{(L)}$ and $\xi_n^{(R)}$ vanishes. The metric on the group manifold, which is constructed using the one-forms R_n , by definition is left-invariant with the Killing vectors $\xi_n^{(L)}$.

The group space of $SU(2)$ group is isomorphic to one of the “remarkable” spheres S^0, S^1, S^3 and S^7 , which are characterized by the left \times right parallelism.

The vector fields on the sphere S^3 are related with the angular momentum operator as

$$L_n^{(R)} = -i\xi_n^{(R)}, \quad L_n^{(L)} = i\xi_n^{(L)}.$$

It follows from the relation (A.7) that the components of the operator of angular momentum satisfy the usual commutation relation, which does not distinguish between left and right rotations:

$$[L_n, L_m] = i\varepsilon_{nmk} L_k.$$

Eigenfunctions of the operator of angular momentum are known as *Wigner functions*

$$D_{m\mu}^l(\varphi, \theta, \psi) \equiv e^{im\varphi} d_{m\mu}^l(\theta) e^{i\mu\psi},\tag{A.7}$$

where $d_{m\mu}^l(\theta)$ are defined as [10]:

$$\begin{aligned}d_{m\mu}^l(\theta) &= \left(\frac{(l-m)!(l+m)!}{(l-\mu)!(l+\mu)!} \right)^{\frac{1}{2}} (1-x)^{\frac{m+\mu}{2}} (1+x)^{-\frac{m-\mu}{2}} \\ &\times P_{l+m}^{(-m-\mu, -m+\mu)}(x),\end{aligned}\tag{A.8}$$

$x = \cos \theta$ and $P_n^{(a,b)}(x)$ is a Jacobi polynomial

$$P_n^{(a,b)}(x) = \frac{(-1)^n}{2^n n!} (1-x)^{-a} (1+x)^{-b} \frac{d^n}{dx^n} \left[(1-x)^{a+n} (1+x)^{b+n} \right].$$

The Wigner function is related to the generalized spherical harmonics as

$$Y_{\mu l m}(\theta, \varphi) = D_{\mu m}^l(-\varphi, \theta, \varphi).$$

The matrices (A.2), which correspond to the fundamental representation of the group $SU(2)$, are particular cases of the Wigner functions: $D_{\mu m}^{1/2}(\varphi, \theta, \psi) = U(\varphi, \theta, \psi)$.

However, the difference between left and right rotations on the sphere S^3 , which is hidden behind the general definition of the operator of angular momentum, reappears if we consider the ladder operators $L_{\pm} = L_1 \pm L_2$. Then the Wigner functions satisfy the equations

$$\begin{aligned} L_{\pm}^{(R)} D_{m\mu}^l &= \sqrt{l(l+1) - \mu(\mu \pm 1)} D_{m\mu \pm 1}^l, & L_3^{(R)} D_{m\mu}^l &= \mu D_{m\mu}^l; \\ L_{\pm}^{(L)} D_{m\mu}^l &= -\sqrt{l(l+1) - m(m \mp 1)} D_{m \mp 1 \mu}^l, & L_3^{(L)} D_{m\mu}^l &= -m D_{m\mu}^l. \end{aligned} \tag{A.9}$$

B Quaternions

Four-dimensional Euclidean space \mathbb{R}^4 is quite special, since it admits a natural multiplicative structure. This becomes very important in clarifying the description of the moduli spaces of the monopoles. In this Appendix, we briefly give addition material to that used in Sect. 6.5.1.

Let us consider a set of 2×2 complex matrices \mathcal{R}^4 . It is closed under matrix addition and multiplication by real scalars and, therefore, may be considered as a real vector space. The bases of the space \mathcal{R}^4 are given by the set of matrices

$$\begin{aligned} e_1 &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \mathbb{I}_2, & e_2 &= \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix} = -i\sigma_1, \\ e_3 &= \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = -i\sigma_2, & e_4 &= \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} = -i\sigma_3, \end{aligned} \quad (\text{B.1})$$

which satisfy the algebra

$$e_4 e_\mu = e_\mu e_4 = e_\mu, \quad e_n e_m = -\delta_{nm} + \varepsilon_{nmk} e_k \quad (n, m, k = 1, 2, 3). \quad (\text{B.2})$$

The basis $\{e_\mu\}$ provides a natural isomorphism from \mathcal{R}^4 to \mathbb{R}^4 given by the mapping

$$X = e_1 x_1 + e_2 x_2 + e_3 x_3 + e_4 x_4 \rightarrow x_\mu = (x_1, x_2, x_3, x_4).$$

Since the basis $\{e_\mu\}$ is orthonormal, this mapping does not change the norm $\|X\|^2 = x_1^2 + x_2^2 + x_3^2 + x_4^2$ and such an isomorphism is an isometry. Note that a matrix X of the space \mathcal{R}^4 can be written as

$$X = \begin{pmatrix} x_1 - ix_4 & -ix_2 - x_3 \\ -ix_2 + x_3 & x_1 + ix_4 \end{pmatrix}, \quad (\text{B.3})$$

and then the norm $\|X\|^2 = \det X$.

The commutation relations (B.2) is a particular case of the so-called *algebra of quaternions*. The space of quaternions \mathcal{H} can be viewed as the set of complex matrices \mathcal{R}^4 equipped with a standard set of matrix operations, or as the vector space \mathbb{R}^4 with multiplicative structure.

Since e_1 is a multiplicative identity, we can drop it and write an arbitrary quaternion as $X = x_0 + x_n e_n$. The operation of the quaternionic conjugation is defined as

$$X \rightarrow \bar{X} = x_\mu \bar{e}_\mu = x_0 - x_n e_n.$$

Thus, if a quaternion X is considered as a matrix in \mathcal{R}^4 , its conjugated \bar{X} is the conjugated transpose matrix.

The product of two quaternions can be computed using the relations (B.2). In particular, we have $\bar{X}X = X\bar{X} = \|X\|^2$ and $\overline{XY} = \bar{Y}\bar{X}$. The real and imaginary parts of a quaternion are

$$\operatorname{Re} X = \frac{1}{2}(X + \bar{X}) = X_4, \quad \operatorname{Im} X = \frac{1}{2}(X - \bar{X}) = X_n e_n.$$

Quaternions whose imaginary part is equal to zero are called real quaternions. A *unit quaternion* satisfies the relation $\|X\|^2 = 1$. Clearly, these quaternions correspond to the elements of \mathcal{R}^4 with unit determinant, that is, the group of unit quaternions is actually the group $SU(2)$. Its group space, a sphere S^3 naturally arises as a subspace of \mathbb{R}^4 .

The quaternionic notions make many relations compact and transparent. For example, Euclidean Dirac matrices γ_μ simply become

$$\gamma_\mu = \begin{pmatrix} 0 & e_\mu \\ \bar{e}_\mu & 0 \end{pmatrix}, \quad \{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}.$$

Since the set of unit quaternions forms the group $SU(2)$, the transformation properties of vectors and spinors can also be written in quaternionic notations. Recall that the transformations of the $SU(2)$ group can be decomposed into left and right rotations as $SU(2)_L \times SU(2)_R$. The unit quaternions X and Y can be set into correspondence with elements of these subgroups: $X \rightarrow x \in SU(2)_L$, $Y \rightarrow y \in SU(2)_R$. Then a vector quaternion transforms as $v \rightarrow Xv\bar{Y}$, while the spinor quaternions s, c , which correspond to representations of the Lorentz group $(0, \frac{1}{2})$, $(\frac{1}{2}, 0)$, respectively, transform as

$$s \rightarrow Xs; \quad c \rightarrow Yc.$$

In this notation, the Euclidean Dirac equation for a massless spinor reads

$$\begin{pmatrix} 0 & D \\ \bar{D} & 0 \end{pmatrix} \begin{pmatrix} s \\ c \end{pmatrix} = 0,$$

where $D \equiv e_\mu D_\mu$ is the quaternionic Dirac operator and $\bar{D} = \bar{e}_\mu D_\mu$. It is decoupled into a pair of Weyl equations that describe the massless fermion of a given chirality:

$$Dc = 0, \quad \bar{D}s = 0.$$

Note that the operator $\bar{D}D = D_\mu D_\mu$ is the usual Laplace operator.

The (pseudo)-scalar and (pseudo)-tensor quaternions may be constructed by multiplication of the spinor and vector quaternions. Let us take, for example, two vectors $v = v_\mu e_\mu$ and $w = w_\mu e_\mu$. One easily finds that the real parts of the quaternionic products $v\bar{w}$ and $\bar{v}w$ transform like scalars while their

imaginary parts transform like a self-dual and anti-self-dual antisymmetric tensor of second rank, respectively. In particular, the quaternionic equation

$$\bar{D}v = 0,$$

which defines the vector of tangent space T_v , can be written in component notation as

$$D_\mu v_\mu = 0, \quad D_\mu v_\nu - D_\nu v_\mu = \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} D_\rho v_\sigma. \quad (\text{B.4})$$

The second of these equations is a self-duality equation for the tensor $F_{\mu\nu} = D_\mu v_\nu - D_\nu v_\mu$.

C $SU(2)$ Transformations of the Monopole Potential

Let us consider the transformations that relate the monopole potential in the Abelian gauge and the hedgehog gauge, respectively. On the spatial asymptotic, the potential of the non-Abelian $SU(2)$ monopole becomes

$$A_n = A_n^a \frac{\sigma^a}{2} = \varepsilon_{amn} \frac{r_m}{er^2} \frac{\sigma^a}{2} = -\frac{1}{er^2} [\mathbf{r} \times \mathbf{T}]_n, \quad (\text{C.1})$$

where the isospin operator is taken in the fundamental representation of the $SU(2)$ group: $T^a = \frac{1}{2}\sigma^a$. Cartesian components of the monopole vector potential are:

$$A_x = \frac{1}{2re} \begin{pmatrix} -\sin \theta \sin \varphi & -i \cos \theta \\ i \cos \theta & \sin \theta \sin \varphi \end{pmatrix}, \quad (\text{C.2})$$

$$A_y = \frac{1}{2re} \begin{pmatrix} \sin \theta \cos \varphi & -\cos \theta \\ -\cos \theta & -\sin \theta \cos \varphi \end{pmatrix}, \quad A_z = \frac{\sin \theta}{2re} \begin{pmatrix} 0 & ie^{-i\varphi} \\ -ie^{i\varphi} & 0 \end{pmatrix}, \quad (\text{C.3})$$

where we used the standard parameterization in terms of azimuthal and polar angles

$$x = r \sin \theta \cos \varphi, \quad y = r \sin \theta \sin \varphi, \quad z = r \cos \theta.$$

Clearly, the non-Abelian magnetic field, which corresponds to the potential (C.1), is regular everywhere in \mathbb{R}^3 but the origin $\{0\}$:

$$B_n = B_n^a \frac{\sigma^a}{2}, \quad B_n^a = \frac{1}{2} \varepsilon_{nmk} F_{mk}^a = \frac{r^a r_n}{er^4}, \quad (\text{C.4})$$

where the field strength tensor is

$$F_{mn}^a = \partial_m A_n^a - \partial_n A_m^a - e \varepsilon_{abc} A_m^b A_n^c. \quad (\text{C.5})$$

The matrix of $SU(2)$ transformations, which unwraps the ‘‘hedgehog’’ from the spherically symmetric form (C.1) to the third axis, is

$$U(\theta, \varphi) = e^{-i(\sigma\hat{\varphi})\theta/2} = e^{-i\sigma_3 \frac{\varphi}{2}} e^{-i\sigma_2 \frac{\theta}{2}} e^{i\sigma_3 \frac{\varphi}{2}} = \begin{pmatrix} \cos \frac{\theta}{2} & -\sin \frac{\theta}{2} e^{-i\varphi} \\ \sin \frac{\theta}{2} e^{i\varphi} & \cos \frac{\theta}{2} \end{pmatrix}. \quad (\text{C.6})$$

This transformation also rotate the Pauli matrices as

$$U^{-1}\sigma_k U = (\cos\varphi\hat{\theta}_k - \sin\varphi\hat{\varphi}_k)\sigma_1 + (\cos\varphi\hat{\varphi}_k + \sin\varphi\hat{\theta}_k)\sigma_2 + \hat{r}_k\sigma_3.$$

However, this transformation is singular at the south pole $\theta = \pi$. To understand the situation better, let us define a regularized polar angle

$$\Theta = \theta \frac{1 + \cos\theta}{1 + \cos\theta + \varepsilon^2},$$

where the parameter ε removes the singularity [49, 131]. Then, the regularized matrices $\tilde{U} = U(\Theta, \varphi)$ rotate the monopole potential as

$$\begin{aligned}\tilde{U}^{-1}A_x\tilde{U} &= \frac{\sin\varphi}{2er} \begin{pmatrix} \sin(\Theta-\theta) & [\cos(\Theta-\theta) - i\cos\theta\cot\varphi]e^{-i\varphi} \\ [\cos(\Theta-\theta) + i\cos\theta\cot\varphi]e^{i\varphi} & -\sin(\Theta-\theta) \end{pmatrix}, \\ \tilde{U}^{-1}A_y\tilde{U} &= -\frac{\cos\varphi}{2er} \begin{pmatrix} \sin(\Theta-\theta) & [\cos(\Theta-\theta) + i\cos\theta\tan\varphi]e^{-i\varphi} \\ [\cos(\Theta-\theta) - i\cos\theta\tan\varphi]e^{i\varphi} & -\sin(\Theta-\theta) \end{pmatrix}, \\ \tilde{U}^{-1}A_z\tilde{U} &= \frac{i\sin\theta}{2er} \begin{pmatrix} 0 & e^{-i\varphi} \\ -e^{i\varphi} & 0 \end{pmatrix},\end{aligned}\tag{C.7}$$

and the affine part of the gauge transformation is

$$\begin{aligned}-\frac{i}{e}\tilde{U}^{-1}\partial_x\tilde{U} &= -\frac{\sin\varphi}{er\sin\theta} \begin{pmatrix} \sin^2\frac{\Theta}{2} & \frac{1}{2}\sin\Theta e^{-i\varphi} \\ \frac{1}{2}\sin\Theta e^{i\varphi} & -\sin^2\frac{\Theta}{2} \end{pmatrix} \\ &\quad + \frac{i\Theta'}{2er}\cos\theta\cos\varphi \begin{pmatrix} 0 & e^{-i\varphi} \\ -e^{i\varphi} & 0 \end{pmatrix}, \\ -\frac{i}{e}\tilde{U}^{-1}\partial_y\tilde{U} &= \frac{\cos\varphi}{er\sin\theta} \begin{pmatrix} \sin^2\frac{\Theta}{2} & \frac{1}{2}\sin\Theta e^{-i\varphi} \\ \frac{1}{2}\sin\Theta e^{i\varphi} & -\sin^2\frac{\Theta}{2} \end{pmatrix} \\ &\quad + \frac{i\Theta'}{2er}\cos\theta\sin\varphi \begin{pmatrix} 0 & e^{-i\varphi} \\ -e^{i\varphi} & 0 \end{pmatrix}, \\ -\frac{i}{e}\tilde{U}^{-1}\partial_z\tilde{U} &= -\frac{i\Theta'}{2er}\sin\theta \begin{pmatrix} 0 & e^{-i\varphi} \\ -e^{i\varphi} & 0 \end{pmatrix},\end{aligned}\tag{C.8}$$

where

$$\Theta' = d\Theta/d\theta = \frac{1 + \cos\theta}{1 + \cos\theta + \varepsilon^2} \left(1 + \theta\varepsilon^2 \frac{1 - \sin\theta}{1 + \cos\theta + \varepsilon^2} \right)$$

is singular in the limit $\varepsilon^2 \rightarrow 0$.

Thus, the smoothed gauge transformation of the $SU(2)$ monopole potential on the spatial asymptotic

$$A_n^{\text{Abelian}} = \frac{1}{2}A_n^{\text{Str}}\sigma^a = \tilde{U}^{-1}A_n\tilde{U} - \frac{i}{e}\tilde{U}^{-1}\partial_n\tilde{U},$$

gives the regularized form of the potential in the Abelian gauge A_n^{Abelian} :

$$A_n^{\text{Abelian}} = -\frac{1}{2er} \left\{ \hat{\varphi}_n \left(\frac{\cos \Theta - 1}{\sin \theta} + \sin(\Theta - \theta) \right) \sigma_3 \right. \quad (\text{C.9}) \\ \left. + \left[\left(\cos(\Theta - \theta) - \frac{\sin \Theta}{\sin \theta} \right) \hat{\varphi}_n \sigma_1 + (\Theta' - 1) \hat{\theta}_n \sigma_2 \right] \begin{pmatrix} e^{i\varphi} & 0 \\ 0 & e^{-i\varphi} \end{pmatrix} \right\}$$

or

$$A_n^{\text{Abelian}} = \frac{1}{er} \left\{ \left[\frac{1 - \cos \Theta}{\sin \theta} - \sin(\Theta - \theta) \right] \hat{\varphi}_n \delta_{a3} \right. \\ - \left[(1 - \Theta') \hat{\theta}_n \sin \varphi + \left(\cos(\Theta - \theta) - \frac{\sin \Theta}{\sin \theta} \right) \hat{\varphi}_n \cos \varphi \right] \delta_{a1} \\ \left. + \left[(1 - \Theta') \hat{\theta}_n \cos \varphi - \left(\cos(\Theta - \theta) - \frac{\sin \Theta}{\sin \theta} \right) \hat{\varphi}_n \sin \varphi \right] \delta_{a2} \right\}.$$

The same transformation \tilde{U} of the non-Abelian magnetic field $B_n = B_n^a \sigma^a / 2$, where B_n^a is given by (C.4), yields

$$B_n \rightarrow \tilde{U}^{-1} B_n \tilde{U} = \frac{r_n}{2er^3} \begin{pmatrix} \cos(\Theta - \theta) & -\sin(\Theta - \theta) e^{-i\varphi} \\ -\sin(\Theta - \theta) e^{i\varphi} & -\cos(\Theta - \theta) \end{pmatrix},$$

that is

$$B_n^a \rightarrow -\frac{r_n}{er^3} (\delta_{a3} \cos(\Theta - \theta) - \sin(\Theta - \theta) (\delta_{a1} \cos \varphi + \delta_{a2} \sin \varphi)).$$

In the naive limit $\Theta \rightarrow \theta$, we would obviously recover the Coulomb field of the Abelian monopole without any singular pieces, while the potential (C.9) would take the form of a Dirac monopole potential embedded into $SU(2)$ group:

$$A_n^{\text{Abelian}} \rightarrow \frac{1}{2er} \frac{1 - \cos \theta}{\sin \theta} \hat{\varphi}_n \sigma_3. \quad (\text{C.10})$$

However, the singularity at $\theta = \pi$ requires more careful treatment. Indeed, although the isotopic components A_n^1, A_n^2 of the non-Abelian vector potential vanish as we take the limit $\Theta \rightarrow \theta$, they still contribute to the field strength tensor F_{mn}^a . Indeed, the third component of the non-Abelian magnetic field (C.5) is

$$B_n^3 = \varepsilon_{nmk} \partial_m A_k^3 - e \varepsilon_{nmk} A_m^1 A_k^2. \quad (\text{C.11})$$

Clearly, the differentiation of the singular Dirac potential at the first term here produces not only Coulomb magnetic field we expected, but also a singular flux of the Dirac string (see (1.49)):

$$B_n = \frac{r_n}{er^3} - \frac{4\pi}{e} \hat{z} \theta(-z) \delta(x) \delta(y). \quad (\text{C.12})$$

The non-Abelian nature of the potential we are considering modifies this result, because the contribution of the second term in (C.11) is also non-zero. Indeed, the piece

$$\Delta B_n = \lim_{\varepsilon^2 \rightarrow 0} e \varepsilon_{nmk} A_m^1 A_k^2 = \lim_{\varepsilon^2 \rightarrow 0} \frac{r_n}{e r^3} (1 - \Theta') \left(\cos(\Theta - \theta) - \frac{\sin \Theta}{\sin \theta} \right)$$

does not vanish at $\theta = \pi$ due to the singularity of the derivative Θ' .

Let us consider it at the vicinity of this point as a distribution on the volume measure $r^2 \sin \theta d\theta d\varphi$. Then the non-vanishing contribution of ΔB_n takes the form

$$\begin{aligned} \lim_{\delta \rightarrow 0} \lim_{\varepsilon^2 \rightarrow 0} r^2 \int_{\pi-\delta}^{\pi} \sin \theta d\theta \int_0^{2\pi} d\varphi \Delta B_n &= -\frac{2\pi \hat{z}}{e} \lim_{\delta \rightarrow 0} \lim_{\varepsilon^2 \rightarrow 0} \int_{\pi-\delta}^{\pi} d\theta \Theta' \sin \theta \\ &= \frac{2\pi \hat{z}}{e} \lim_{\delta \rightarrow 0} \lim_{\varepsilon^2 \rightarrow 0} \cos \Theta(\theta) \Big|_{\pi-\delta}^{\pi} = \frac{4\pi}{e} \hat{z}. \end{aligned} \quad (\text{C.13})$$

Therefore,

$$\Delta B_n = \frac{4\pi}{e} \hat{z} \theta(-z) \delta(x) \delta(y),$$

which precisely cancels the string singularity of the field of the Dirac monopole. Thus, the field of the $SU(2)$ monopole contains no singularity in the Abelian gauge [131].

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